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OPTIMIZATION OF BENDING BEAM FOR PRECISE POSITIONING OF SPR SENSORS

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ABSTRACT

Piezoelectric cantilevers have been in great demand in actuator applications for positioning purposes. The tip displacement of the cantilever is the most important and sensitive parameter in the actuation mode. Hence, in this study a bimorph cantilever beam has been chosen as the element of study consisting of two active piezoelectric layers of PVDF bonded together. An attempt has been made to establish the expression of tip deflection of a bimorph cantilever beam of different piezoelectric. This bimorph cantilever is designed and the results are simulated using COMSOL Multiphysics software. The analytical results and simulated data are found to be in close agreement with each other, hence validating the expression of tip deflection of a bimorph cantilever beam.

KEYWORDS: Piezoelectric effect, actuator..

INTRODUCTION

Piezoelectric transducers have attracted significant attention due to their simple and relative ease of implementation into a wide range of applications as compared to conventional ones such as electrostatic, magnetic and thermal transducers etc (Anton, 2011; Oiu J and Ji H, 2010; Fu Y, 2005). The beams, diaphragms or cantilevers are the most commonly employed structures for piezoelectric transduction. Among them, the cantilevers are the most preferred and studied structures used as sensors, transducers, switches and relays etc (Sepaniak M et al., 2002; Vashist SK, 2007; Hansen KM et al., 2001; Goeders KM et al., 2008). These cantilevers have been realized in different configurations such as unimorph, bimorph and multimorph structures depending on the required flexural motion and sensitivities (Lee SY, Ko B and Yang W, 2005).

There are two basic modes in which these cantilevers can be operated based on the required output parameter. It is called as static mode if the measured output is the structure deformation and dynamic mode if the quantity measured is the resonant frequency of the cantilever (Lang HP et al., 2005). In the static mode, the displacement of the cantilever beam depends on the type of loading, dimensions of the structure, spring constant, modulus of elasticity etc. However, in its dynamic mode, a shift in the resonant frequency of cantilever is a measure of sensing. These two operating modes require different design parameters of cantilever. The design constraints require the cantilever to be long and deformable for its operation in static mode whereas short and stiff cantilevers are preferred for its working in dynamic mode (Salehi-Khojin A, 2008).

In this work, a bimorph cantilever in its static mode is considered as the study element (Priya S, 2009) Most of the studies on the bimorph cantilevers so far have been limited to the two bonded piezoelectric layers being equal in length. It is, therefore, of interest, to investigate a bimorph cantilever whose piezoelectric layers are of unequal lengths. The purpose of this study is to determine and validate the effect of the ratio of upper and lower piezoelectric lengths of a piezoelectric bimorph cantilever on its tip deflection.

USE OF COMSOL MULTIPHYSICS AND BOUNDARY CONDITIONS

A piezoelectric bimorph cantilever beam is designed and its electrical response is simulated and studied with COMSOL Multiphysics software (Version 4.2) based on finite element modeling (Norouzi M and Kashaninia A, 2009; Hutton DV, 2004; Pryor RW, 2011). It consists of two active piezoelectric layers of Polyvinylidene difluoride (PVDF) bonded together. These layers are poled along the direction of thickness. The length, width and thickness of the beam are along 1st, 2nd and 3rd direction respectively as shown in figure 1.

The model uses a piezoelectric application module for the simulation of mechanical and the electrical behavior of the piezoelectric cantilever beam (Smits JG, Dalke SI and Cooney TK, 1991). In this model, one end of the cantilever beam is clamped in accordance with mechanical boundary conditions. Hence, the vertical faces of the cantilever at one end are constrained to move. However the unconstrained faces are free to bend along the direction of applied electric field/ force and due to the load of the beam itself. The effects of the electrodes were not considered in the geometry because their mechanical behavior can be neglected due to their thickness. The voltage of 100 volts is applied along the interface, while the top and bottom faces of upper and lower PVDF layers are grounded. Zero charge/ Symmetry constraint was applied on the other faces.

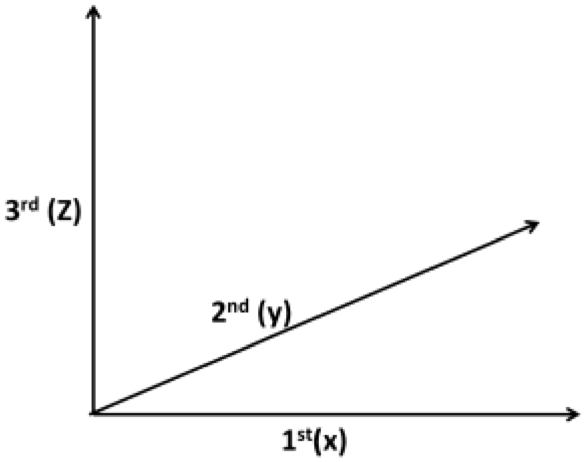

Figure 1. The chosen directions of length, width and thickness of cantilever beam.

An electric field is generated along the Z direction and the same is directed towards the interface from both the ends of the bimorph cantilever beam. Consequently, one layer expands while the other contracts along the length, which results in the deflection of the cantilever, along the Z direction. The material properties of PVDF are taken from the material library of COMSOL.

Theoretical analysis of bimorph cantilever

Let us consider the length and the width of the two piezoelectric PVDF layers are kept same but the thicknesses of the two layers are varied as shown schematically in figure 2.

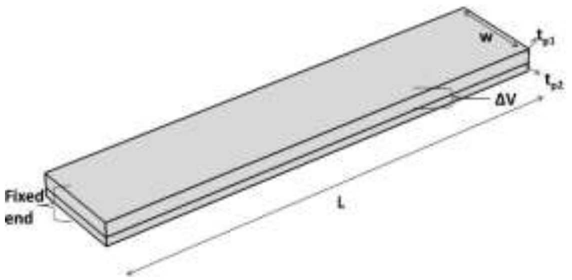

Figure 2. A schematic diagram of a bimorph actuator consists of two piezoelectric layers of different thickness.

Hence, for an applied electric field, the tip deflection $\left(\delta\right)$ of the bimorph cantilever can be written as (Huang C, Lin YY and Tang TA, 2004):

$$\delta = \frac{3E_{p1}E_{p2}(t_{p1}+t_{p2})^2 d_{31}\Delta VL^2}{4E_{p1}E_{p2}t_{p1}t_{p2}(t_{p1}+t_{p2})^2 + (E_{p1}t_{p1}{}^2 - E_{p2}t_{p2}{}^2)^2} \qquad (1)$$

where ΔV is the potential difference between the interface of the two PVDF layers and the upper face of the upper piezoelectric layer or lower face of the bottom piezoelectric layer, t_{p1} and E_{p1} are the thickness and elastic modulus of the upper piezoelectric layer respectively, t_{p2} and E_{p2} are the thickness and elastic modulus of the bottom piezoelectric layer respectively and d_{31} is the piezoelectric coupling coefficient relates strain in 1st direction to the electric field in 3rd direction and L is the total length of the beam.

This relation is applicable when the bimorph cantilever is operated in parallel mode. The tip deflection is found to be larger for parallel mode of operation as compared to series mode (Huang C, Lin YY and Tang TA, 2004) and hence, in this work, the operation of bimorph cantilever is considered for parallel mode only.

The above equation (1) was valid when the lengths of the two piezoelectric active layers are equal and cannot be applied to bimorphs having unequal lengths. It is seen that the length ratios of piezoelectric and non-piezoelectric layers in unimorph cantilevers have profound effect on its deflection (Gao X, Shih WE and Shih WY, 2009). Hence, an effort has been made to establish the relation of tip deflection with different length ratios of the active piezoelectric layers in case of a bimorph cantilever.

Let us first consider a unimorph cantilever with unequal lengths of piezoelectric and non-piezoelectric layers as shown in figure 3(a)-3(c).

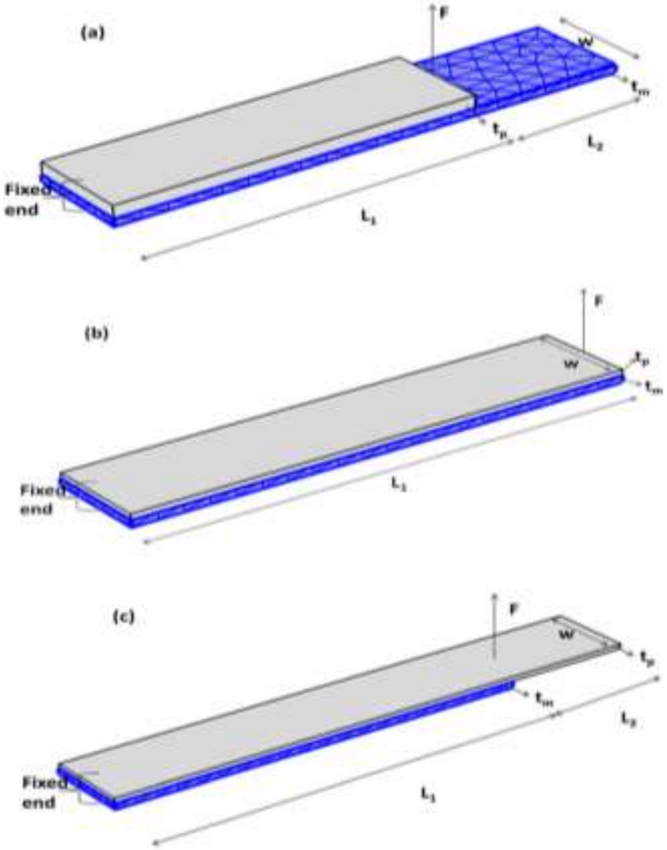

Figure 3(a)-(c). A schematic diagram of a unimorph actuator beam with a nonpiezoelectric to piezoelectric length ratio (a)>1, (b) =1 and (c) <1.

A unimorph cantilever beam with unequal lengths of piezoelectric and nonpiezoelectric layers can be divided into two sections $'L_1'$ and $'L_2'$ where $'L_1'$ is the section that consists of both piezoelectric and non-piezoelectric layers of the beam and $'L_2'$ is the section that consists only of either piezoelectric layer or non-piezoelectric layer .The effect of the various possible length ratios on the tip displacement of a unimorph cantilever are discussed below(Gao X, Shih WE and Shih WY, 2009).

(i) When nonpiezoelectric layer is longer than piezoelectric layer, the tip deflection of the beam is given by (Gao X, Shih WE and Shih WY, 2009).

$$\delta = \frac{1}{6}\left[\frac{F}{wD_2}L_1^3 - \frac{3F(L_1+L_2)}{wD_2}L_1^2 - \left(\frac{1}{D_1}-\frac{1}{D_2}\right)\frac{3FL_1}{w}(L_1+2L_2)L_1 + \left(\frac{1}{D_1}-\frac{1}{D_2}\right)\frac{FL_1^2(L_1+3L_2)}{w}\right] \tag{2}$$

Where

$L_1 + L_2$ and w are the total length and width of the cantilever respectively.

D_1 and D_2 are the bending modulus per unit width and can be expressed as (Shen Z, Shih WY and Shih WH, 2006; Li X et al.,1999):

$$D_1 = \frac{E_m^2 t_m^4 + E_p^2 t_p^4 + 2E_m E_p t_m t_p (2t_m^2 + 2t_p^2 + 3t_m t_p)}{12(E_m t_m + E_p t_p)} \tag{3}$$

$$D_2 = \frac{1}{12}E_m t_m^3 \tag{4}$$

Here t_m and E_m are the thickness and elastic modulus of non piezoelectric layer respectively, t_p and E_p are the thickness and elastic modulus of piezoelectric layer respectively.

F is the output force obtained due to the converse piezoelectric action and can be expressed as equivalent to blocking force required to compensate the converse piezoelectric action. Hence, the relation of F can be written as (Wang QM and Cross LE, 1998):

$$F = \frac{3wE_m t_m E_p t_p (t_m + t_p)}{4L_1(E_m t_m + E_p t_p)} d_{31} E_3 \tag{5}$$

(ii) When the lengths of two layers are equal to each other i.e . for $L_2 = 0$, the tip deflection is expressed as:

$$\delta = \frac{1}{6}\left[\frac{F}{wD_2}L_1^3 - \frac{3FL_1^3}{wD_2} - \left(\frac{1}{D_1}-\frac{1}{D_2}\right)\frac{3FL_1^3}{w} + \left(\frac{1}{D_1}-\frac{1}{D_2}\right)\frac{FL_1^3}{w}\right] \tag{6}$$

(iii) When nonpiezoelectric layer is shorter than the piezoelectric layer, the tip deflection of the beam is given by equation (2), however in this case the expression for D_2 changes to:

$$D_2 = \frac{1}{12} E_p t_p^3 \tag{7}$$

This analytical approach can be extended to the bimorph actuator consisting of two piezoelectric layers of unequal lengths as shown in figure 4(a)-(c).

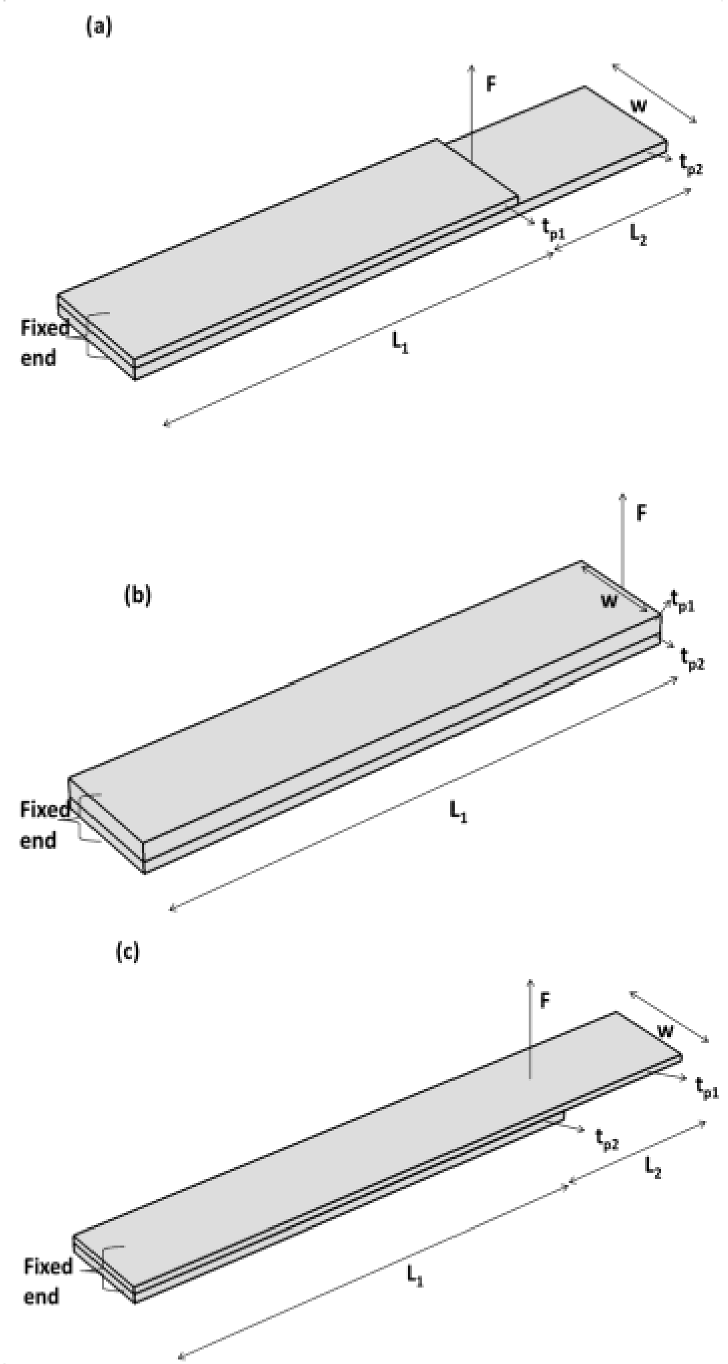

Figure 4(a)-(c). A schematic diagram of a bimorph actuator beam with a nonpiezoelectric to piezoelectric length ratio (a)>1, (b) =1 and (c) <1.

Similar to the unimorph actuators, the deflection for all possible configurations of unequal piezoelectric lengths of bimorphs can be obtained by the above relations and are summarized below:

(i) When the lower piezoelectric layer is longer than the upper piezoelectric layer, the tip deflection of the beam is given by the equation (2) but in this case:
$E_m = 0$, So

$$D_1 = \frac{1}{12} E_p t_{p1}^3 \tag{8}$$

$$D_2 = \frac{1}{12} E_p t_{p2}^3 \tag{9}$$

Where t_{p1} and t_{p2} are the thickness of upper and lower piezoelectric layer respectively.

The blocking force for a bimorph cantilever can be written as (Li X et al.,1999):

$$F = \frac{3w(t_{p1} + t_{p2})^2 E_p}{8L_1} d_{31} E_3 \tag{10}$$

(ii) When the lengths of two piezoelectric active layers are equal to each other, the tip deflection for a bimorph actuator is obtained using equations (1),(8),(9) and (10) with $L_2 = 0$ .It can be seen that when $L_2 = 0$, the tip deflection obtained from equation (1) and (2) are equal to each other with thickness as a varying parameter.

(iii) When the lower piezoelectric layer is shorter than the upper piezoelectric layer, the tip deflection of the beam is same as given in case (i) with:

$$D_2 = \frac{1}{12} E_p t_{p1}^3 \tag{11}$$

RESULTS AND DISCUSSIONS

The variation in the tip deflection with the thickness of the piezoelectric layer of a bimorph cantilever beam is shown in figure 5. The analytical results obtained are depicted in the figure by circles and the results obtained through the simulation method using COMSOL Multiphysics software are marked with squares.

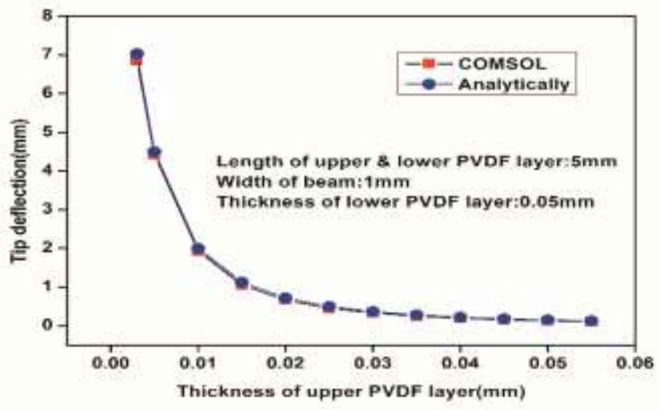

Figure 5. Variation in the tip deflection of the bimorph actuator with the thickness of the PVDF layer.

The simulation and analytical results are seen to be in close agreement with each other. It can be seen from the figure that the thickness of PVDF layer has great impact on the deflection of the beam. It is concluded from the graph that thinner the piezoelectric layer, greater is the tip deflection. With the decrease in the thickness of the piezoelectric layer, the electric field across it increases, for a constant applied potential. Hence, the strain along the length of the piezoelectric layer increases resulting in significant bending deformation of the beam. Therefore, the thickness of the piezoelectric layer can be tuned and optimized depending on the required tip deflection of the beam. It can be seen from the figure that the tip deflection increases drastically if the thickness of the upper PVDF layer is below 0.025mm while the thickness of the lower layer is maintained at 0.005mm. Hence, for the analysis of unequal piezoelectric lengths of bimorph cantilever beam, this particular configuration is considered.

The variation in the tip deflection with different lengths of the piezoelectric layers is shown in figure 6. The simulated and analytical results are found to complement each other. It is seen that the tip displacement increased with the length of variable PVDF layer.

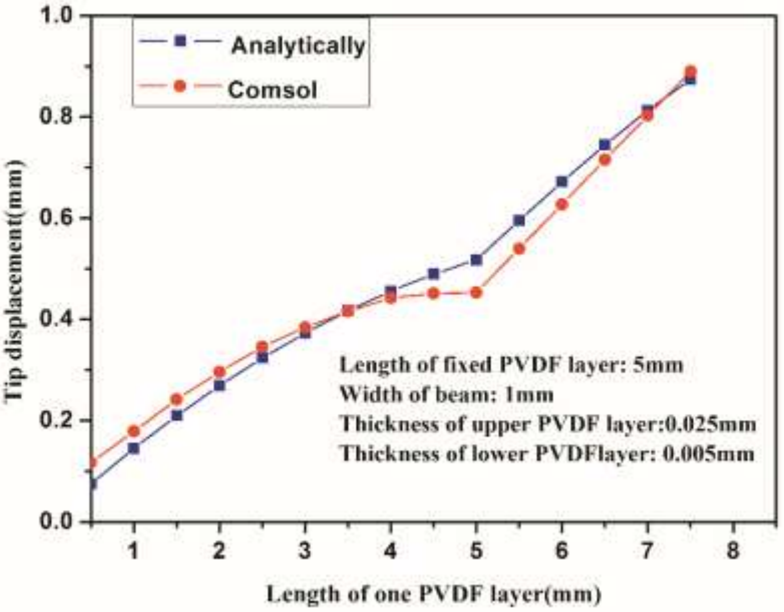

Figure 6. Variation in the tip deflection of the bimorph cantilever beam with the length of a PVDF layer.

The simulated data of variation in the tip displacement with the width of the bimorph cantilever beam is shown in figure 7. It can be seen from figure that the width of the cantilever beam has no significant effect on the tip deflection. It can be seen that the variation in the tip displacement for bimorph cantilever is from 0.43mm to 0.52mm .This is a very small change as compared to the variation in the width of the beam from 1mm to 5mm. This may also be concluded from the analytical relations which shows that the tip deflection is independent of the variation in the width. It is well known that the area moment of inertia of a beam is directly proportional to its width and deflection is inversely proportional to the moment of inertia. Hence an increase in width would result in decrease in the tip deflection. However, an increase in width also results in the increase of the load acting on the beam hence, it increases the tip displacement though no significantly (Negi LS, 2008).

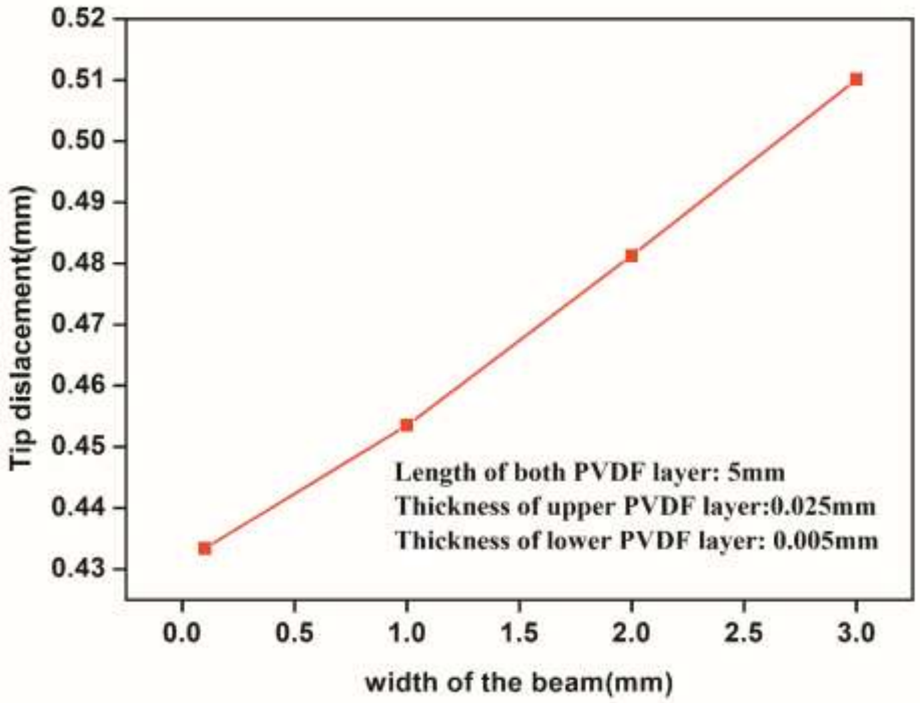

Figure 7. Variation in the tip deflection of the bimorph cantilever beam with the width of the cantilever using COMSOL Multiphysics.

CONCLUSIONS

The relation of the tip displacement for a bimorph cantilever beam with varying piezoelectric lengths has been established based on the Euler- Bernoulli beam theory and validated with the simulated results. The simulated data and theoretical calculations are found to be consistent. These relations are essential and beneficial for optimizing the design parameters of a bimorph actuator before the actual fabrication of the device.

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